wjec cbac

# GCE AS/A Level

0977/01

S17-0977-01

# MATHEMATICS – FP1
## Further Pure Mathematics

FRIDAY, 19 MAY 2017 – MORNING

1 hour 30 minutes

**ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

**INSTRUCTIONS TO CANDIDATES**

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

**INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

**1.** The matrix $\mathbf{M}$ is given by

$$\mathbf{M} \quad \begin{bmatrix} 1 & & \\ 2 & & \\ 3 & & \end{bmatrix}.$$

*(a)* Evaluate the determinant of $\mathbf{M}$. [2]

*(b)* (i) Find the adjugate matrix of $\mathbf{M}$.

(ii) Deduce the inverse matrix $\mathbf{M}^{-1}$. [3]

*(c)* Hence solve the system of equations

$$\begin{bmatrix} 1 & & \\ 2 & & \\ 3 & & \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 11 \\ 17 \end{bmatrix}.$$

[2]

**2.** Consider the series

$$S_n \quad ^2 \quad ^2 \quad ^2 \qquad n \quad ^2.$$

Obtain an expression for $S_n$, giving your answer in the form $an^3 \quad bn^2 \quad cn$, where $a$, $b$, $c$ are rational numbers. [6]

**3.** The complex number $z$ is given by $z = \dfrac{(1+2\mathrm{i})(-3+\mathrm{i})}{(1+3\mathrm{i})}$ .

Determine the modulus and the argument of $z$. [8]

**4.** The transformation $T$ in the plane consists of a reflection in the $x$-axis, followed by a translation in which the point $(x, y)$ is transformed to the point $(x \qquad y \qquad$ , followed by an anticlockwise rotation through 90° about the origin.

*(a)* Show that the matrix representing $T$ is

$$\begin{bmatrix} 0 & & -1 \\ 1 & & -2 \\ 0 & & \end{bmatrix}.$$

[5]

*(b)* Show that $T$ has no fixed points. [3]

**5.** Consider the following equations.

$$\begin{array}{l} x \quad y \quad z \\ x \quad y \quad z \\ x \quad y \quad z \quad \lambda. \end{array}$$

*(a)* Find the value of $\lambda$ for which the equations are consistent. [4]

*(b)* For this value of $\lambda$, find the general solution of the equations. [3]

**6.** Use mathematical induction to prove that $9^n - 1$ is divisible by 8 for all positive integers $n$. [7]

**7.** The function $f$ is defined on the domain $\left(0, \frac{\pi}{2}\right)$ by

$$f(x) = (\tan x)^{\tan x}.$$

*(a)* Show that

$$f'(x) = g(x)\big(1 + 1\text{n}(\tan x)\big),$$

where $g(x)$ is to be determined. [4]

*(b)* Find the $x$-coordinate of the stationary point on the graph of $f$, giving your answer correct to two decimal places. [3]

**8.** The complex numbers $z$ and $w$ are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$wz = 1.$$

*(a)* Obtain expressions for $x$ and $y$ in terms of $u$ and $v$. [4]

*(b)* Given that the point $P$ moves along the line $x + y = 1$,

(i) show that the locus of $Q$ is a circle,

(ii) determine the radius and the coordinates of the centre $C$ of the circle. [6]

*(c)* Given that $P$ and $Q$ have the same coordinates, find the two possible positions of $P$ and $Q$. [3]

**9.** The roots of the cubic equation $x^3 + 2x^2 + 3x + 4 = 0$ are denoted by $\alpha, \beta, \gamma$.

*(a)* (i) Show that

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} = -\frac{7}{16}.$$

(ii) What does this result tell you about the nature of the roots of this cubic equation? [5]

*(b)* Determine the cubic equation whose roots are $\dfrac{\alpha}{\beta\gamma}, \dfrac{\beta}{\gamma\alpha}, \dfrac{\gamma}{\alpha\beta}$. [7]

**END OF PAPER**